Observation of the Goos-Hänchen Shift with Neutrons

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PNCMI, July 7, 2010

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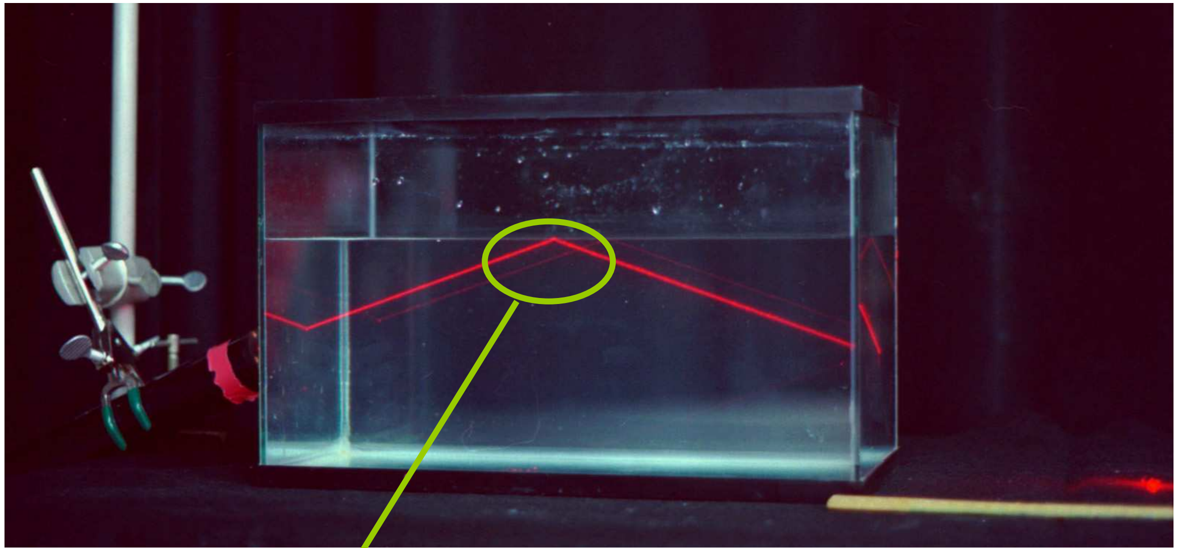
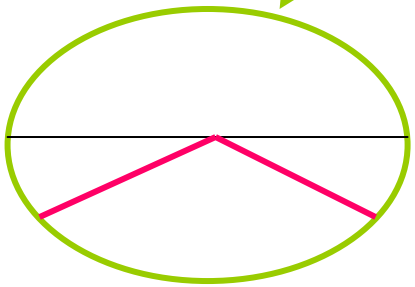
Goos-Hänchen shift

total reflection for light

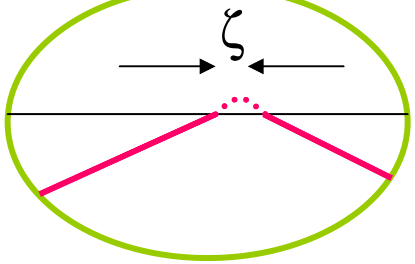


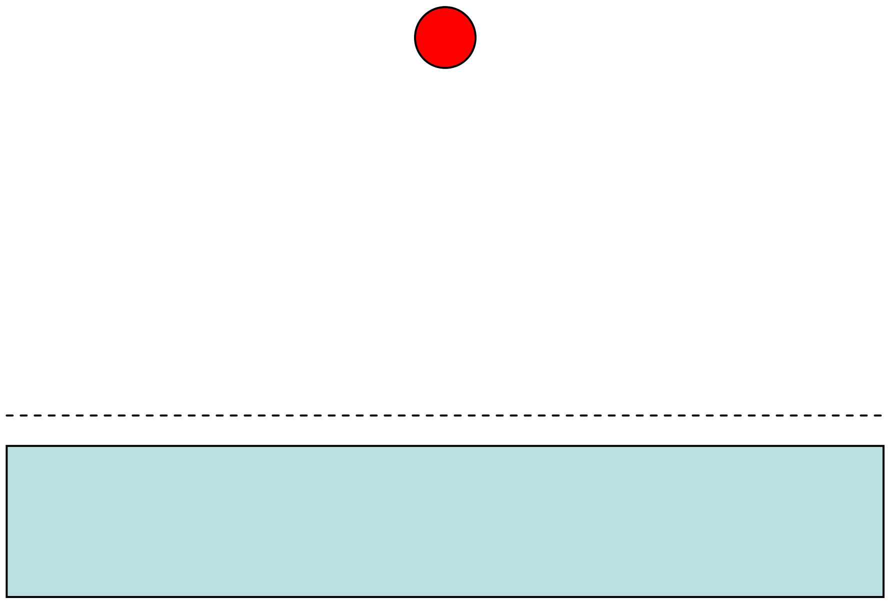
Goos-Hänchen shift ζ
up to 2 μm

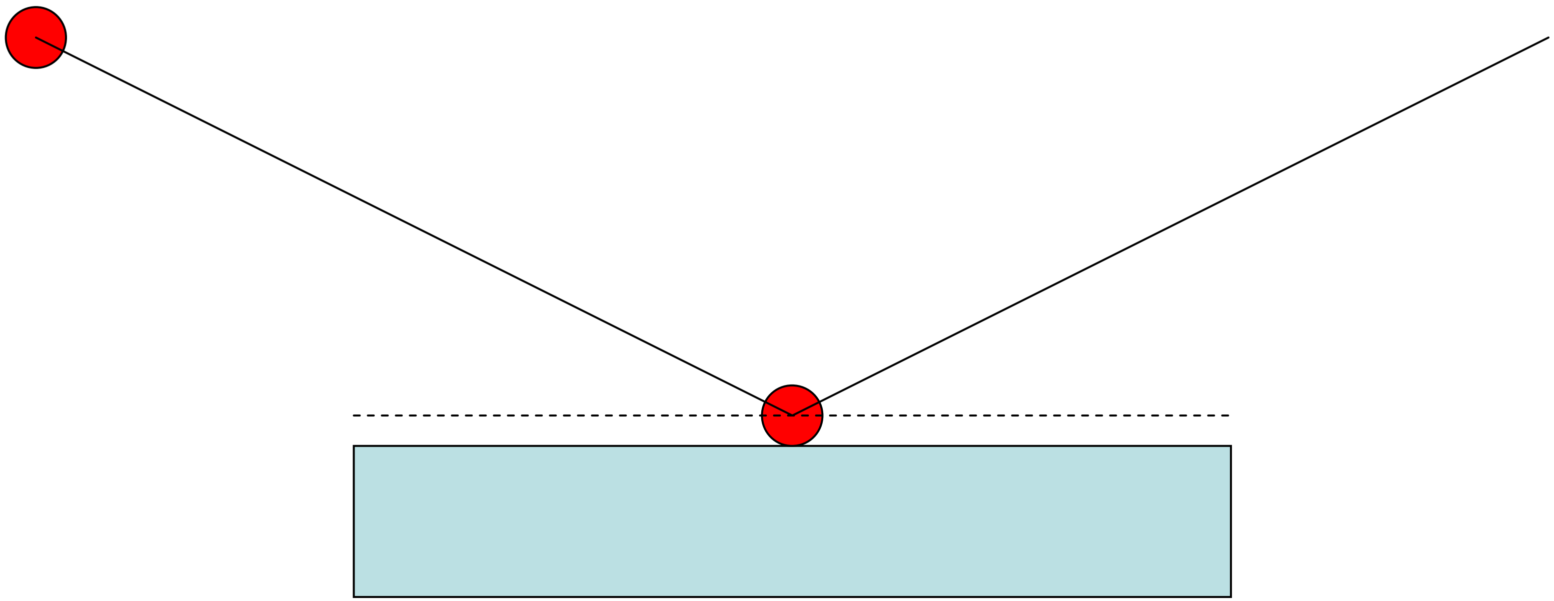


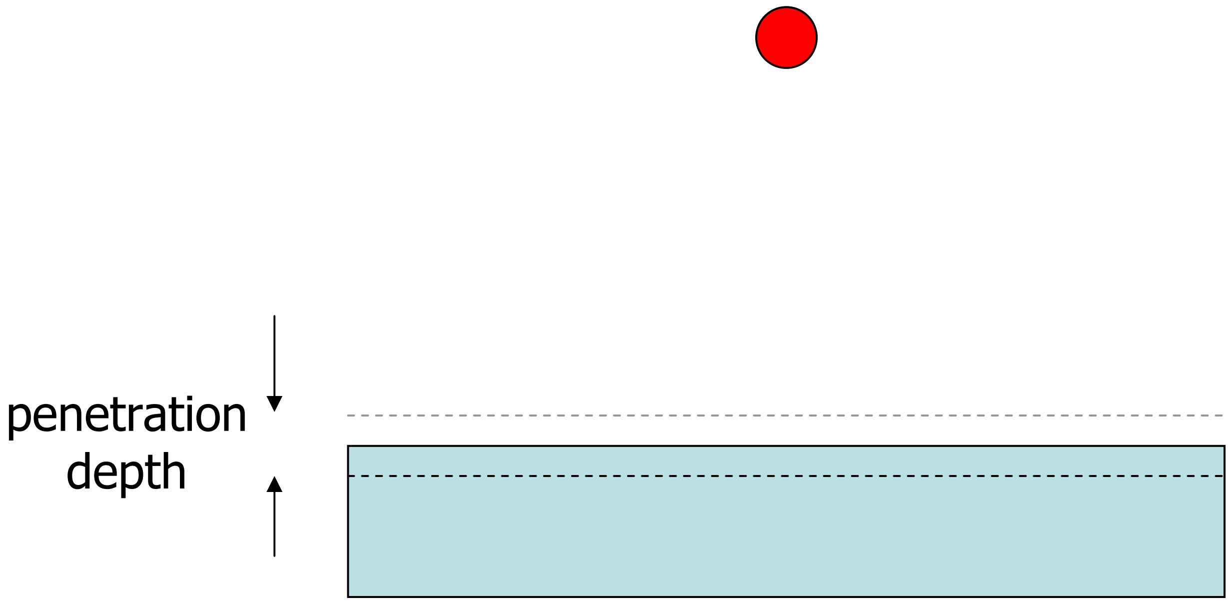
prediction:
I. Newton (~1700)

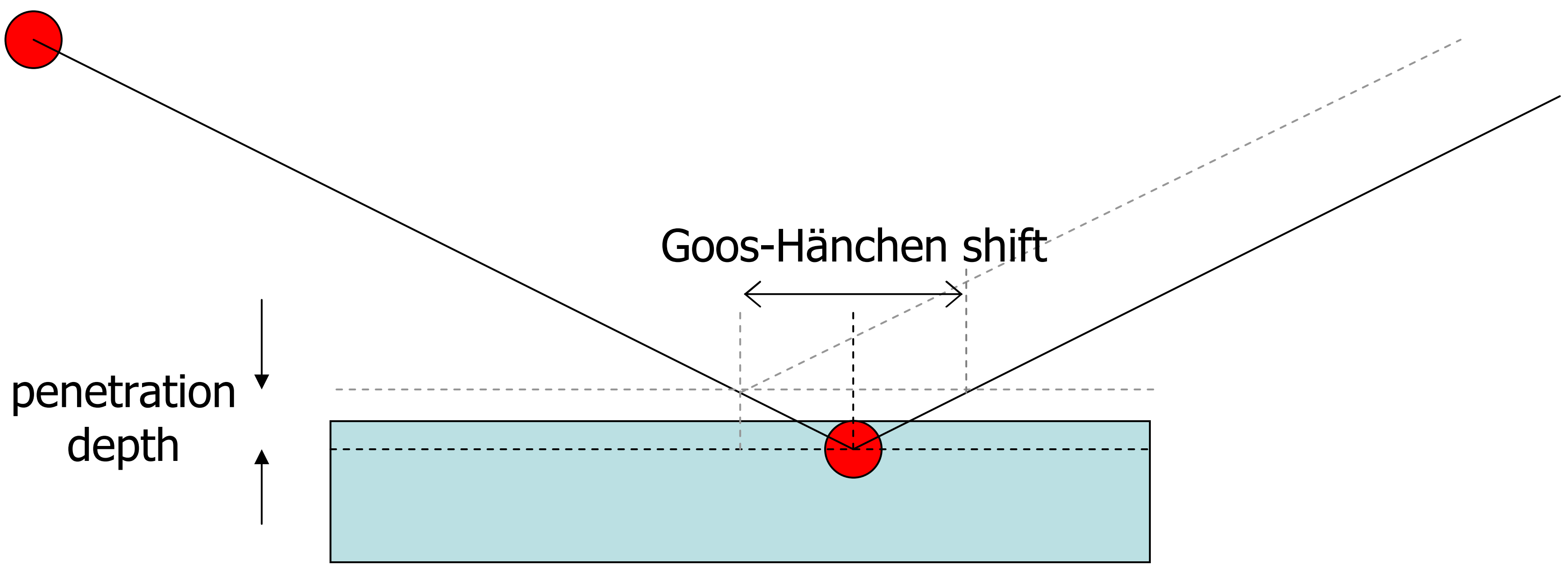
experiment:
F. Goos and H. Hänchen (1949)








penetration
depth

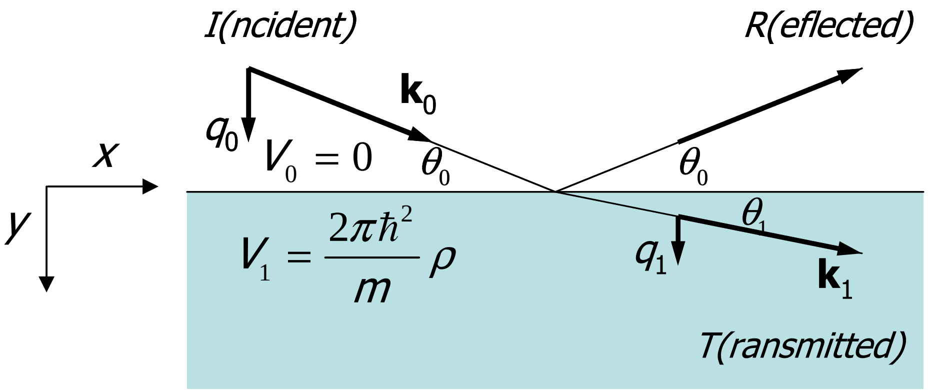

Goos-Hänchen shift
penetration depth

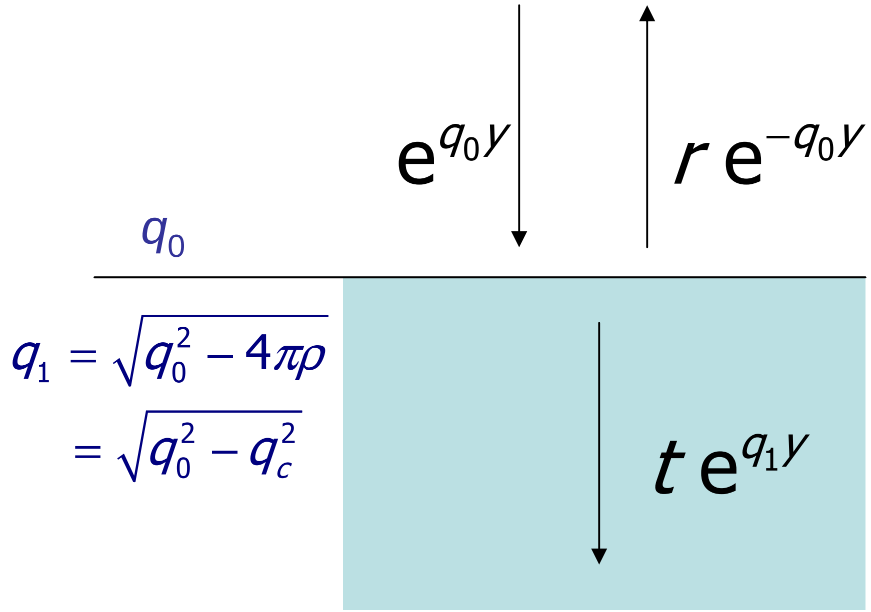
Menu

- GH-shift for particles, theoretical background
  - evanescent wave
- Use of the neutron's magnetic moment
  - different GH-shift for + and – state
  - measuring technique: Larmor precession
- Experiments and results

Neutron reflection at sharp interface (Fresnel)

no magnetic field / sample





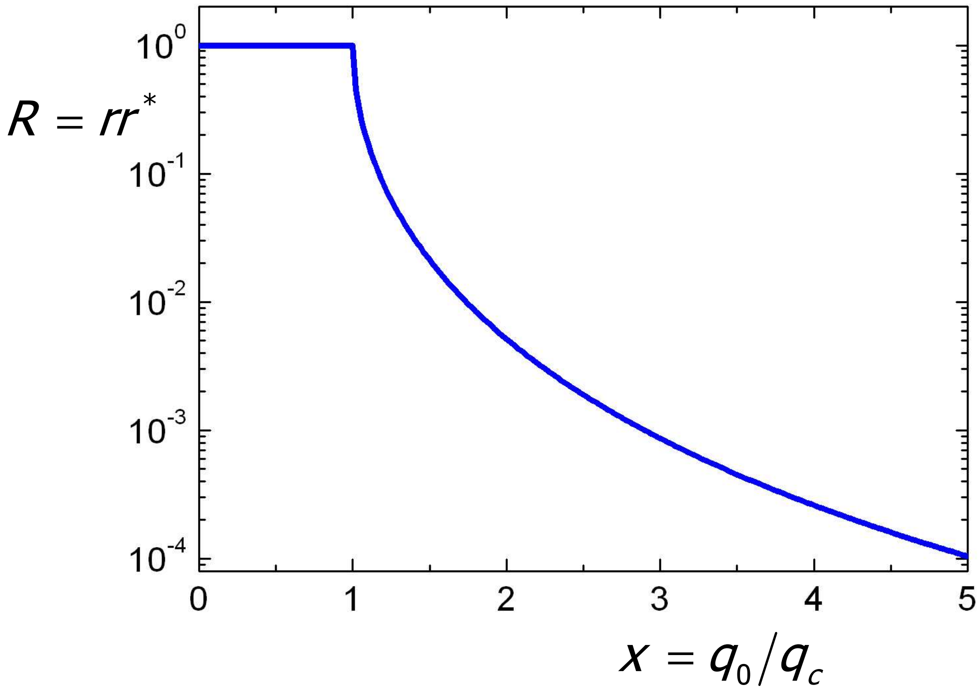
- isotropic in $x - z$
- 1-dim Schrödinger Eq.
- x-component remains unchanged

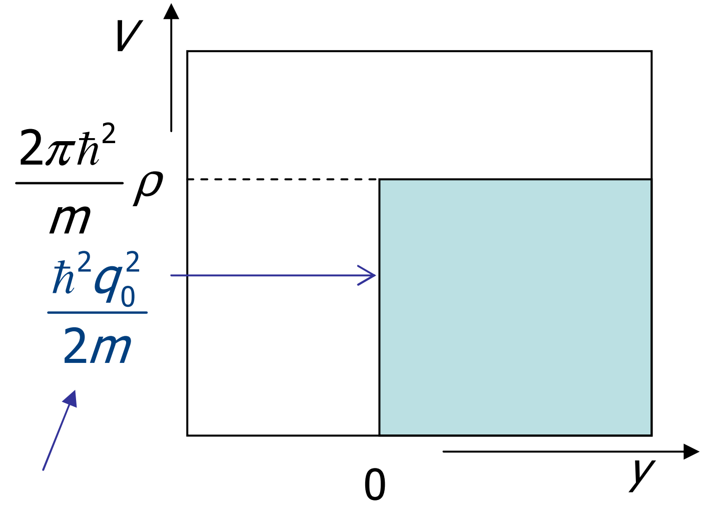
reflection amplitude $r = \frac{q_0 - q_1}{q_0 + q_1}$

transmission amplitude $t = \frac{2q_0}{q_0 + q_1}$

Sharp interface (Fresnel)

$$q_1 = \sqrt{q_0^2 - 4\pi\rho}$$
$$= \sqrt{q_0^2 - q_c^2}$$

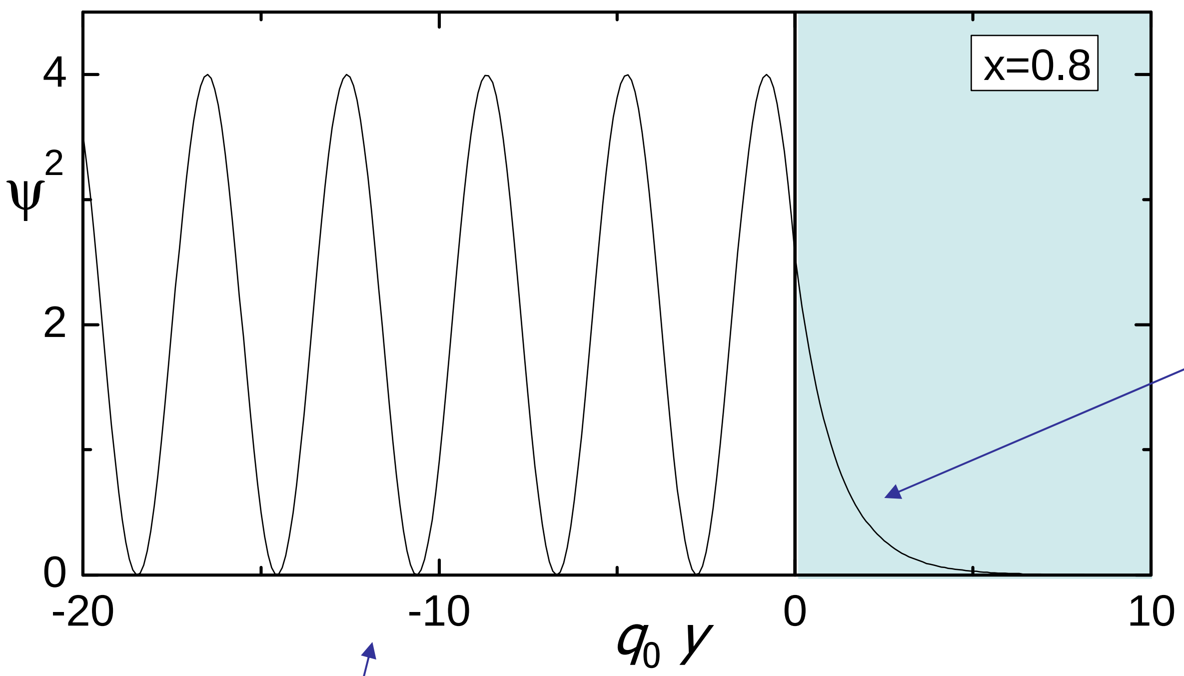




Perpendicular component kinetic energy

Total reflection

$$x = q_0 / q_c$$



$$q_1 = \sqrt{q_0^2 - q_c^2}$$

evanescent wave

$$\psi(y) = \psi(y=0)\, e^{-y/d}$$

penetration depth:

$$q_o d = \left(1 - x^2\right)^{-1/2}$$

$$\psi(y) = e^{iq_0 y} + r e^{-iq_0 y}$$

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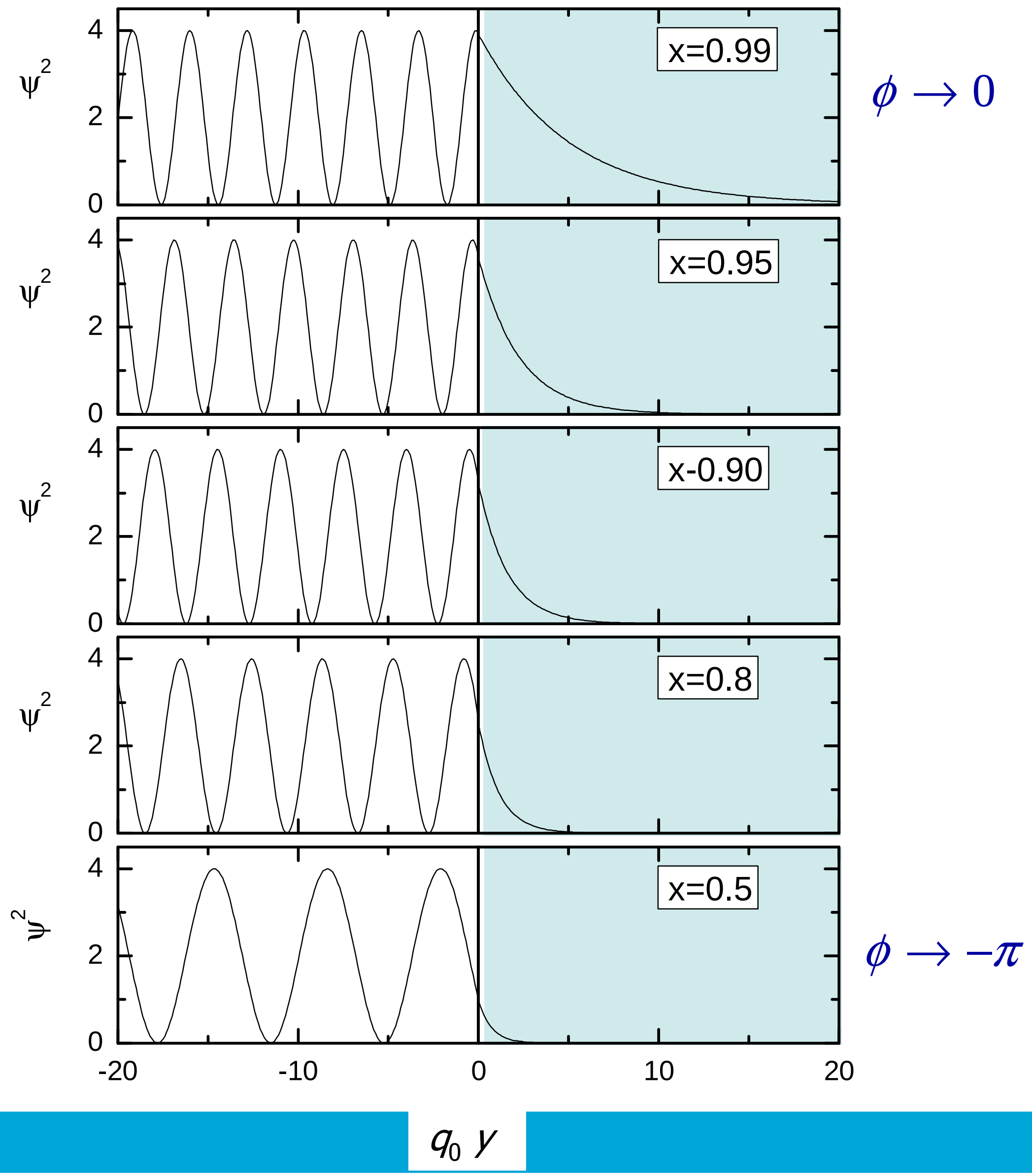
Total reflection

$$\psi(y) = e^{iq_0 y} + re^{-iq_0 y}$$

$$\psi(y=0) = 1 + r = 1 + e^{i\phi}$$

with phase $\phi = -2\arccos(x)$

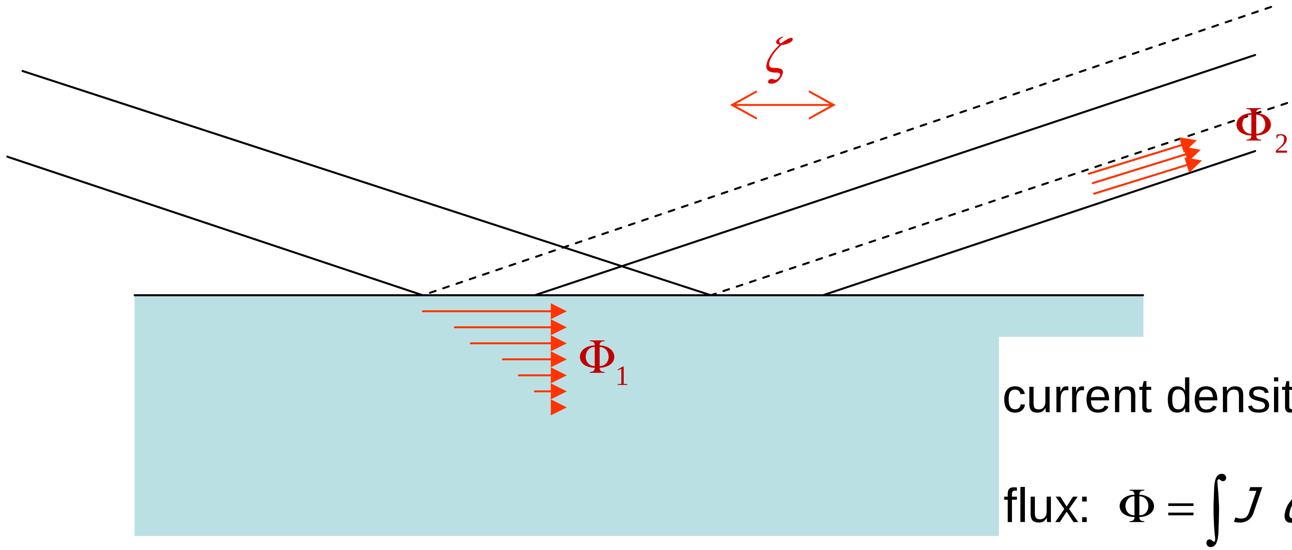
$x = q_0 / q_c$



Unique relation between
- phase
- penetration depth

Goos-Hänchen shift

ref: R.H. Renard, J. Opt. Soc. Am. **54** (1964)1190



current density $\mathbf{J} = \frac{\hbar}{2im}\left(\Psi^{*}\nabla\Psi - \Psi\nabla\Psi^{*}\right)$

flux: $\Phi = \int J \, ds$

conservation of particles: $\Phi_1 = \Phi_2$

leads to shift $$\zeta = \frac{k}{q_c^2}\frac{2q_0}{\sqrt{q_c^2 - q_0^2}}$$

Unique relation between
- phase
- penetration depth
- GH shift

Alternative derivations

V. Ignatovich, Phys. Lett. A **322** (2004) 36

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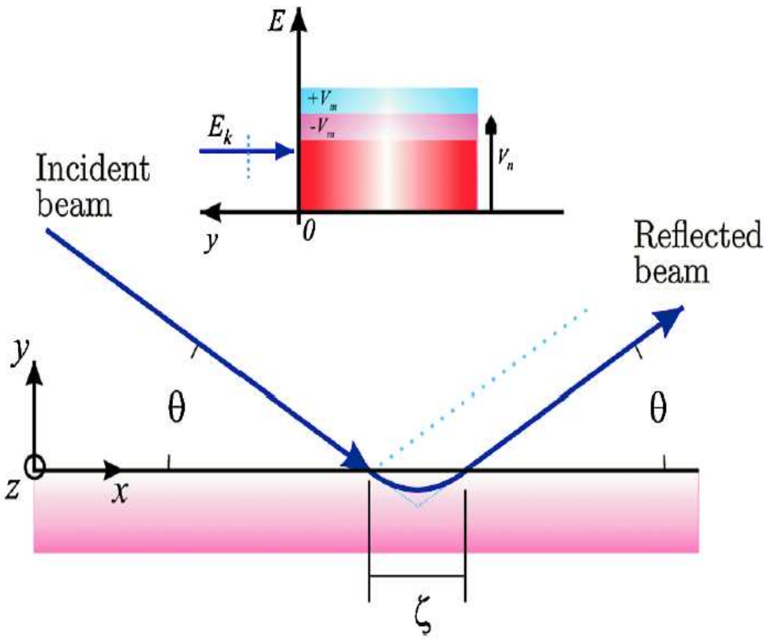
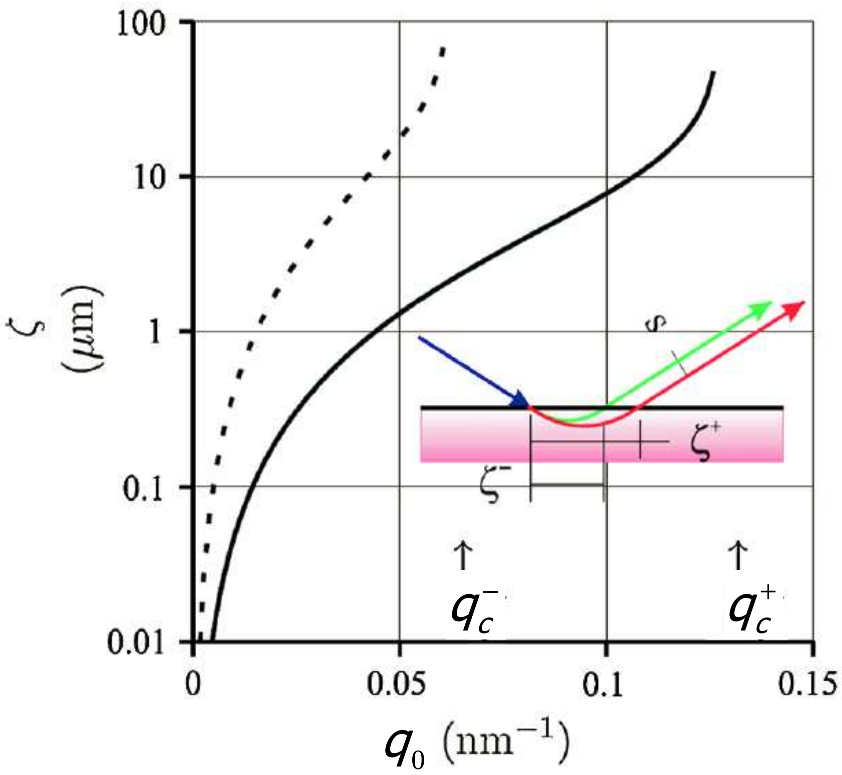
Polarized neutrons and magnetic material



different for + and - state



example: magnetized iron

How to measure it ⇨ Larmor precession

Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial y^2} + \begin{bmatrix} \mu B & o \\ 0 & -\mu B \end{bmatrix} \Psi$$

If the magnetic field is entered at $y = 0$ its solution at $y = L$ is

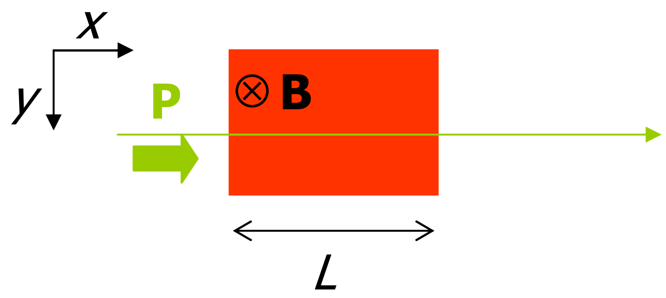
$$\Psi = \begin{bmatrix} \psi^+ \\ \psi^- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i((k_0-\Delta k)L-\omega_0 t)} \\ e^{i((k_0+\Delta k)L-\omega_0 t)} \end{bmatrix}$$

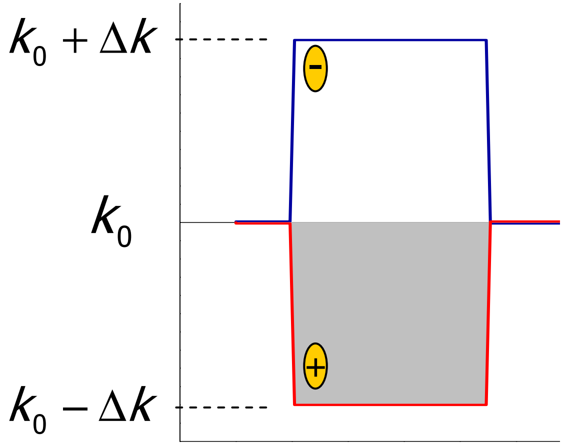
The polarisation is

$$\langle \hat{\sigma}_x \rangle = \frac{\psi^+\psi^{-*} + \psi^-\psi^{+*}}{\psi^+\psi^{+*} + \psi^-\psi^{-*}}$$

$$= \tfrac{1}{2}\left(e^{-2i\Delta kL} + e^{+2i\Delta kL}\right) = \cos 2\Delta k L$$

$$= \cos \frac{2\omega_z L}{v_0}$$





Zeeman energy $\hbar\omega_z = \mu B$

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How to measure it ⇨ Larmor precession

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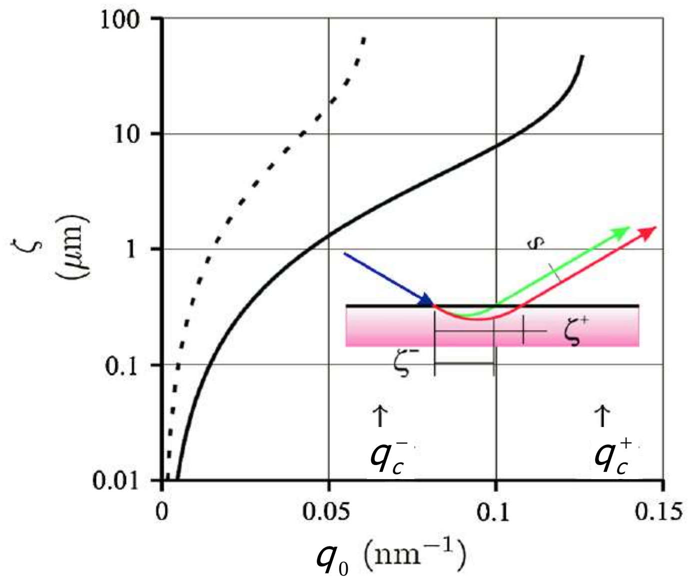

in total reflection region both spin states add different phase to wave function:

Extra 'pseudo' Larmor precession

Zeeman energy $\hbar\omega_z = \mu B$

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How to measure it

Pseudo Larmor precession

total reflection

$$\psi_r^{\pm}(y=0) = r^{\pm} = e^{i\phi^{\pm}}$$

with phase $\phi^{\pm} = -2\arccos\left(q_0/q_c^{\pm}\right)$

$$\Psi_r(y=0) = \begin{bmatrix}\psi_r^{+}(y=0)\\ \psi_r^{-}(y=0)\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\exp(i\phi^{+})\\ \exp(i\phi^{-})\end{bmatrix}$$

$$= \frac{\exp(i\gamma/2)}{\sqrt{2}}\begin{bmatrix}\exp(i\delta/2)\\ \exp(-i\delta/2)\end{bmatrix}$$

with

$$\gamma(q_0) = \phi^{+}(q_0) + \phi^{-}(q_0)$$

$$\delta(q_0) = \phi^{+}(q_0) - \phi^{-}(q_0)$$

analogous to Larmor precession:

$$\langle\hat{\sigma}_x\rangle = \frac{\psi^{+}\psi^{-*} + \psi^{-}\psi^{+*}}{\psi^{+}\psi^{+*} + \psi^{-}\psi^{-*}}$$

$$= \tfrac{1}{2}\left(\exp(+i\delta) + \exp(-i\delta)\right) = \cos\delta$$

$$= \cos\left(\phi^{+}(q_0) - \phi^{-}(q_0)\right)$$

extra precession upon reflection

How to measure it

(i) Spin-echo instrument

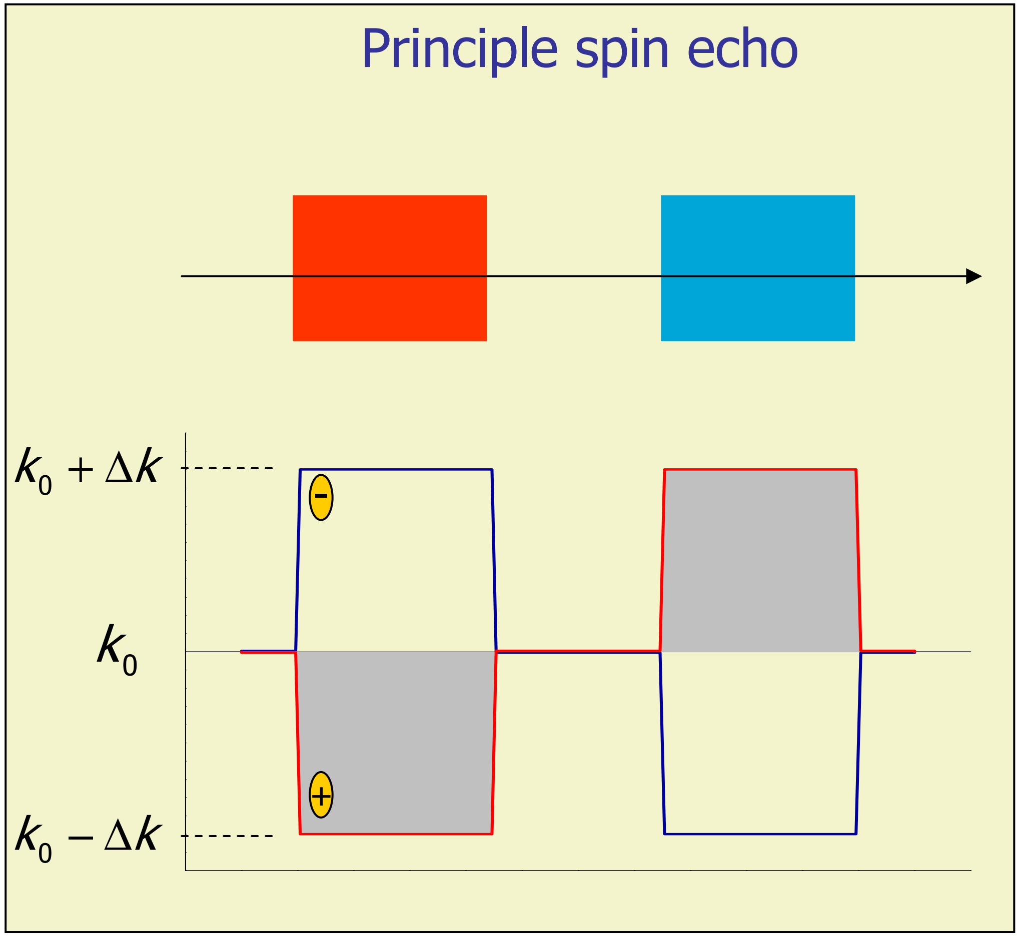
beam polarization
after one precession region:

$$\langle \hat{\sigma}_x \rangle = \psi^+ \psi^{-*} + \psi^- \psi^{+*}$$

$$= \tfrac{1}{2}\left(e^{-2i\Delta kL} + e^{+2i\Delta kL}\right) = \cos 2\Delta kL$$

$$= \cos \frac{2\omega_z L}{v_0}$$

is compensated in 2nd region



How to measure it

(ii) Neutron reflectometer

OffSpec, ISIS, UK

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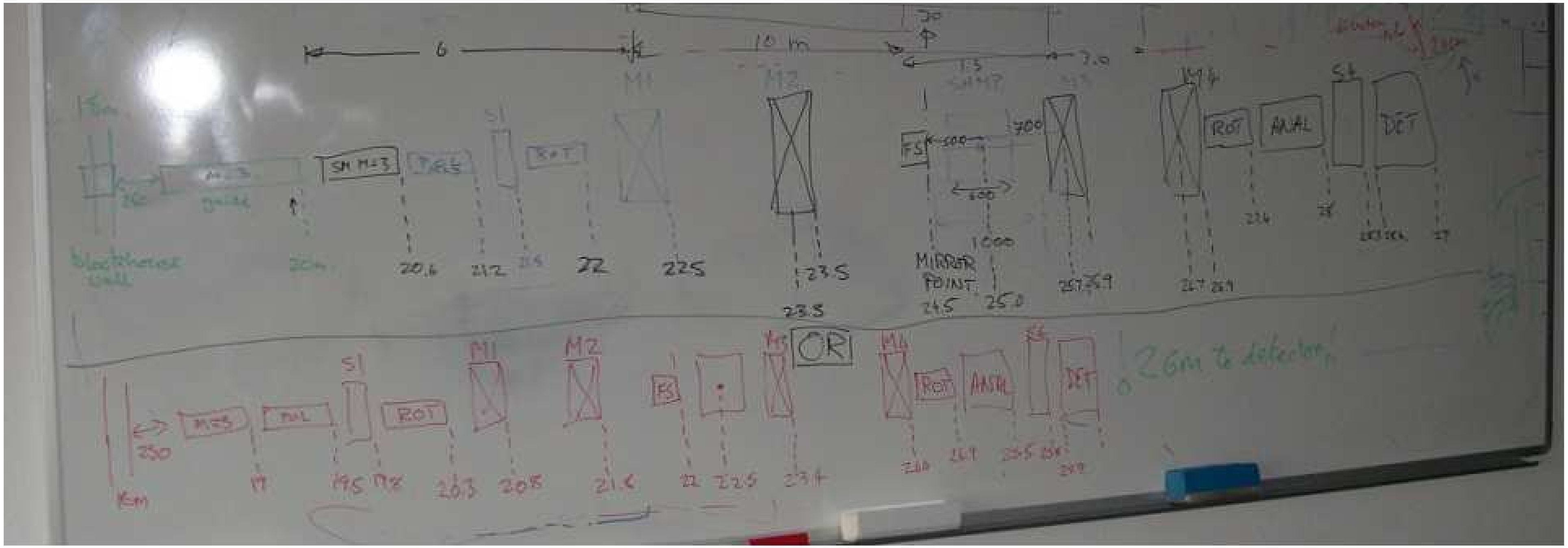
OffSpec, ISIS, UK

collaboration ISIS and Delft University of Technology



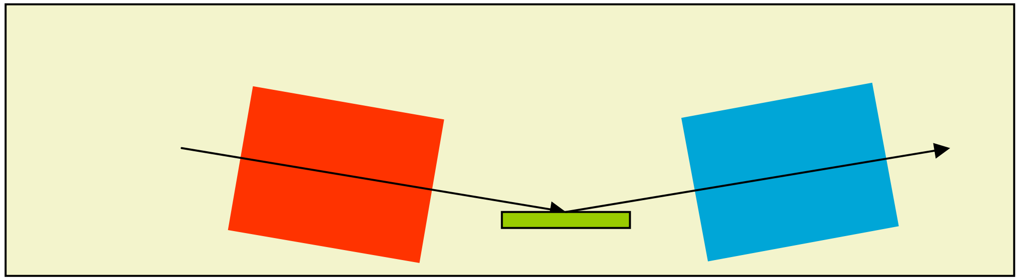
OffSpec, Delft, April 2005

OffSpec, ISIS, UK

collaboration ISIS and Delft University of Technology

OffSpec,
ISIS,
March 2009

Experiment



- sample: Si wafer with 3 μm Permalloy ($Fe_{0.2}Ni_{0.8}$)
  magnetized in plane (**B** ϒ beam)
- OffSpec 'in echo' without sample (measured polarization P_0)
- glancing angle ~ 4 mrad, q_0 scanned by time-of-flight
- two measurements: single and double reflection
- measured spin-echo signal $\dfrac{P}{P_0} = \cos\left(N\delta\left(q_0\right)\right)$

with $\delta\left(q_0\right) = \phi^+\left(q_0\right) - \phi^-\left(q_0\right)$ the Larmor 'pseudo precession' due to different phases at reflection

de Haan *et al.*, PRL **104** (2010) 010401

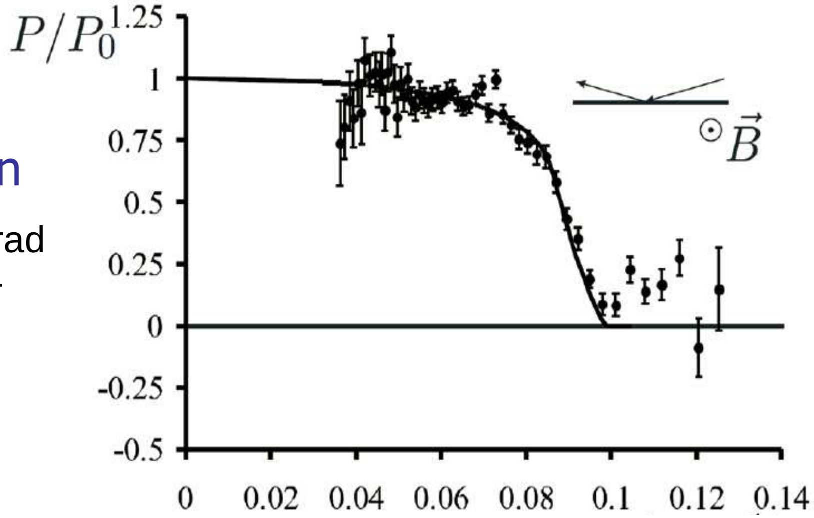
Experiment

Results

black line: theory
red line: theory, with small correction in P_0

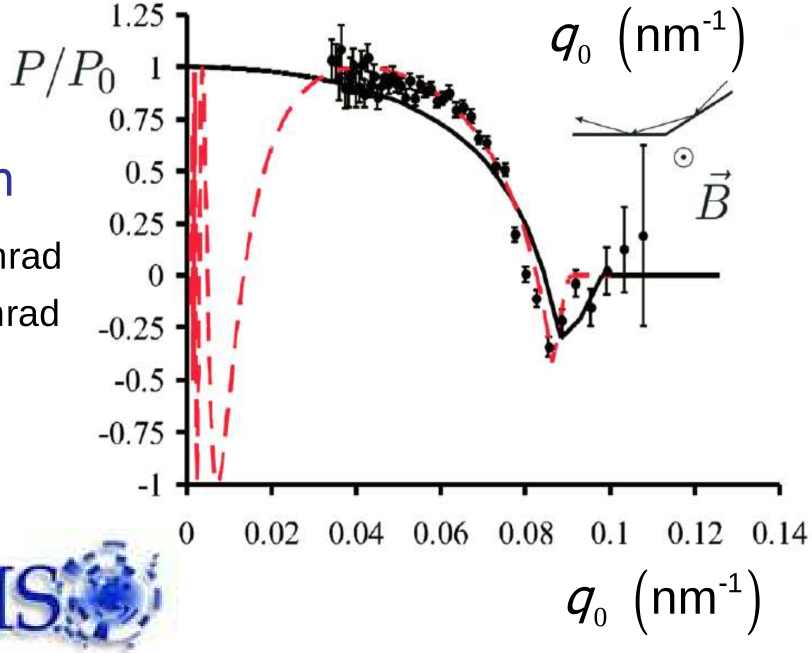
single reflection

$\theta_0 = 5.0$ mrad
$B_s = 1.2$ T

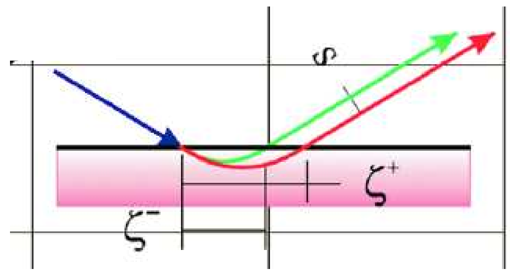


double reflection

$\theta_0 = 4.05$ mrad
$\theta_1 = 3.75$ mrad
$B_s = 1.2$ T



GH shift



$q_0 = 0.06$ nm^{-1} $\zeta^+ = 2.4$ µm
$\zeta^- = 1.0$ µm

$q_0 = 0.09$ nm^{-1} $\zeta^+ = 20$ µm
$\zeta^- = 2.8$ µm

$s \leq 100$ nm

Conclusions

- Goos-Hänchen shift observed for particles
- Measurement is possible with neutrons because
    - different shift for different spin states
    - resulting 'pseudo Larmor precession' can be determined

acknowledgement NWO SIXTH FRAMEWORK PROGRAMME CAPACITIES nmi3